1–8.
[5]Patankar, S. V., *Numerical Heat Transfer and Fluid Flow*, Hemisphere, New York, 1980.
[6]Leonard, B. P., "A Stable and Accurate Convective Modeling Procedure Based on Quadratic Upstream Interpolation," *Computer Methods in Applied Mechanics and Engineering*, Vol. 19, No. 1, 1979, pp. 59–98.
[7]Chung, B. T. F., and Li, H. H., "Forced Convective Cooling Enhancement of Electronic Modules Through a Double Layer Design," *Journal of Electronic Package*, Vol. 117, No. 1, 1995, pp. 69–74.
[8]Aung, W., Fletcher, L. S., and Sernas, V., "Developing Laminar Free Convection Between Vertical Plates with Asymmetric Heating," *International Journal of Heat and Mass Transfer*, Vol. 15, No. 11, 1972, pp. 2293–2308.
[9]Li, H. H., and Chung, B. T. F., "Mixed Convection in a Vertical Channel with Internally Heated Rectangular Blocks," *Fundamentals of Mixed Convection*, edited by T. Y. Chu and T. S. Chen, American Society of Mechanical Engineers HTD-Vol. 274, 1994, pp. 9–16.

Particle Drag Coefficient in Solid Rocket Plumes

H. F. Nelson* and John C. Fields†
University of Missouri–Rolla, Rolla, Missouri 65401

Nomenclature

A	= particle cross-sectional area, $\pi d^2/4$
a	= speed of sound in gas
C_D	= particle drag coefficient, $2D/(\rho U^2 A)$
$(C_D)_{\rm ave}$	= see Eq. 7
$(C_D)_{\rm DSMC}$	= particle drag coefficient from direct simulation Monte Carlo
D	= drag force
d	= particle diameter
Kn	= Knudsen number
M	= freestream Mach number
Re	= Reynolds number based on particle diameter
S	= molecular speed ratio, $M\sqrt{\gamma/2}$
T_g, T_p	= gas temperature, particle temperature
U	= gas-particle relative velocity
X_i	= mass fraction of gas i
γ	= ratio of gas specific heats
ρ	= gas density

Introduction

THE effect of drag on trajectories of particles in solid rocket plumes can significantly influence convective and radiative heat transfer to the base of the rocket and plume radiation signatures. In this Note we present a comparison of direct simulation Monte Carlo (DSMC) drag coefficients for particles in solid rocket plumes with predictions of four empirical drag correlations currently used in plume codes. Typically, the particles are composed of aluminum oxide and have diameters from 1 to 10 μm. Due to their small size they exist in a rarefied flowfield at altitudes greater than about 40 km. Particle Kn vary from 0.5 to 1000, or larger. The plume gas temperatures are of the order of 2500 K in the core and 200 K at the outer edge. Gas-particle relative velocity can vary by up to ±5000 m/s. Also, the composition of the plume is a mixture of many gases including H_2O, H_2, N_2, O_2, OH, NO, H, O, CO, CO_2, Cl_2, and Cl, as well as others.

Received Feb. 10, 1995; revision received March 10, 1995; accepted for publication March 10, 1995.

*Professor of Aerospace Engineering, Department of Mechanical and Aerospace Engineering and Engineering Mechanics. Associate Fellow AIAA.
†Graduate Student, Department of Mechanical and Aerospace Engineering and Engineering Mechanics.

DSMC Method

The DSMC method was developed by Bird[1–4] for applications to low density flow for Kn ranging from 0.025 to 100. For $Kn < 0.025$ the flow is continuum, for $Kn > 100$ the flow is free molecule. DSMC considers a number of simulated molecules, each representing a large number of real molecules. The motion of the molecules is computed in the region surrounding the particle. Molecular paths between collisions are calculated exactly, but collisions are treated statistically. Molecular motion is stopped while collisions are computed statistically throughout the entire flowfield. Next, the molecules are allowed to move in their new directions with their new velocities for a short time and then held motionless in their new positions, while another collision cycle takes place. The collisions are computed by statistical sampling. Accurate, steady-state values of flowfield properties and body-forces are obtained by continuing the calculations for a long period of time while computing a cumulative average of instantaneous samples taken at time intervals large enough to prevent correlation between successive averages. The drag coefficient is calculated from the DSMC normal and shear forces per unit area on the particle assuming complete accommodation and diffuse reflection.

Drag Correlation Equations

A brief description of each drag correlation considered in this study is provided here for convenience. The correlations are evaluated for specific individual gases in the calculations.

1) *Hermsen*: Hermsen[5] correlated C_D data for slip through free molecule flow at subsonic through supersonic velocities. Hermsen's equation is

$$C_D = 2 + (C_{D0} - 2)\exp[-3.07\sqrt{\gamma}(M/Re)\zeta] + \omega/(M\sqrt{\gamma})\exp[-Re/(2M)] \quad (1)$$

where $C_{D0} = 24[1 + 0.15Re^{0.687}]/Re$, $\zeta = [1 + (12.278 + 0.584Re)Re]/[1 + 11.278Re]$, and $\omega = 1.7\sqrt{T_p/T_g} + 5.6/(M + 1)$.

2) *Henderson*: Henderson[6] developed a drag correlation for continuum, slip, transition, and free molecule flow, which includes effects of gas-particle temperature differences. It consists of three equations. One for subsonic flow, one for supersonic flow ($M > 1.75$), and a bridging equation between the subsonic and supersonic regimes, valid for $1.0 < M < 1.75$. For subsonic flow, C_D is given by

$$C_D = \frac{24}{Re + S[4.33 + \Phi\exp(-0.247Re/S)]} + [\Psi + 0.1M^2 + 0.2M^8]\exp[-M/(2\sqrt{Re})] + 0.6S[1 - \exp(-M/Re)] \quad (2)$$

where

$$\Phi = \frac{3.65 - 1.53T_p/T_g}{1 + 0.353T_p/T_g}$$

$$\psi = \frac{4.5 + 0.38(0.03Re + 0.48\sqrt{Re})}{1 + 0.03Re + 0.48\sqrt{Re}}$$

For $M > 1.75$, the Henderson correlation is

$$C_D = \frac{0.9 + (0.34/M^2) + 1.86\sqrt{M/Re}[2 + (2/S^2) + (1.058/S)\sqrt{T_p/T_g} - (1/S^4)]}{1 + 1.86\sqrt{M/Re}} \tag{3}$$

The equation for the linearly interpolated drag coefficient for $1.0 < M < 1.75$ is

$$C_D(M) = C_D(1.0) + 4(M - 1.0)[C_D(1.75) - C_D(1.0)]/3 \tag{4}$$

where $C_D(1.0)$ is obtained from Eq. (2) using $M = 1.0$, and $C_D(1.75)$ is obtained using Eq. (3) with $M = 1.75$.

3) *Crowe*: Crowe[7] developed an equation for C_D for spherical particles in rocket nozzles that correlated well with existing experimental data and with theoretically predicted trends for $Re < 100$ and $M < 2$. Both of these conditions are usually satisfied within plumes. The Crowe drag correlation for a sphere is

$$C_D = (24/Re - 2)\exp[-3.07\sqrt{\gamma}(M/Re)g(Re)] + h(M)\exp[-Re/(2M)] + 2 \tag{5}$$

where $g(Re)$ is evaluated from $\log_{10} g(Re) = 1.25\{1 + \tanh[0.77 \log_{10}(Re) - 1.92]\}$ and $h(M) = \{2.3 + 1.7\sqrt{T_p/T_g} - 2.3 \tanh[1.17 \log_{10}(M)]\}/\{M\sqrt{\gamma}\}$.

4) *Carlson*: The Carlson and Hoglund[8] drag equation is intended to correlate experimental data for particle drag in rocket nozzles. It is given as

$$C_D = \left(\frac{24}{Re}\right) \times \frac{(1 + 0.15Re^{0.687})(1 + \exp[-0.427/M^{4.63} - 3.0/Re^{0.88}])}{1 + M[3.82 + 1.28\exp(-1.25Re/M)]/Re} \tag{6}$$

It is important to recall that each of these correlations is empirical and based on curve fits of experimental data. As such, they are only as accurate as the experiments upon which they were based.

5) *Average drag*: It is useful to define a single, average drag correlation to use for comparison purposes as

$$(C_D)_{\text{ave}} = \frac{(C_D)_{\text{Hermsen}} + (C_D)_{\text{Henderson}} + (C_D)_{\text{Crowe}} + (C_D)_{\text{Carlson}}}{4} \tag{7}$$

Figure 1 shows C_D from each drag correlation as a function of relative velocity for a particle in CO_2 or H_2O at $Kn = 0.5$ for $T_p = T_g = 2000$ K. $(C_D)_{\text{ave}}$ is shown as solid squares on a solid line. $(C_D)_{\text{ave}}$ agrees well with diffuse reflection free molecule calculations of C_D.[9] Each correlation predicts approximately the same value for C_D (within 15–20%) for each gas. Moreover, since the drag correlations are empirical correlations based on experimental data (as opposed to theoretical calculations), $(C_D)_{\text{ave}}$ should be viewed as an accurate predictor of particle drag coefficient.

Sensitivity to Particle Temperature

DSMC results for C_D were obtained for $T_p = 1500$, 2000, and 2500 K. They are averaged and compared to the correlations for $T_p = 2000$ K. To ensure that erroneous conclusions would not be reached, each drag equation was examined to determine the effect of changing T_p. The effect of T_p on the drag correlation predictions was found to be very small for $1500 < T_p < 2500$ K. The variation in C_D was less than $\pm 2\%$ from C_D at $T_p = 2000$ K. Full details on this analysis are given in Appendix D of Ref. 9.

Drag Coefficient

Each drag correlation yields similar values for C_D at conditions typically encountered in rocket plume environments as shown in Fig. 1. Therefore, $(C_D)_{\text{ave}}$ is used as the basis for C_D comparisons.

Figures 2 and 3 show C_D vs U for 4-μm-diam particles for four gases (CO, N_2, CO_2 and H_2O) for $Kn = 0.5$, 1, 2, and 10. Figure 2 shows C_D for CO and N_2. Figure 3 shows C_D for CO_2 and H_2O. The lines are $(C_D)_{\text{ave}}$, and the symbols are $(C_D)_{\text{DSMC}}$ results averaged over 3–6 cases (different values of T_p from 1500 to 2500 K) at each Kn and U. The largest standard deviation in $(C_D)_{\text{DSMC}}$ [as percent of the average $(C_D)_{\text{DSMC}}$] was 2.29% for CO, 1.52% for N_2, 5.46% for CO_2, and 1.89% for H_2O, all occurring at $U = 100$ m/s, $Kn = 0.5$. Note that $(C_D)_{\text{ave}}$ is the same for N_2 and CO because they have the same molecular weight and their other properties are similar (i.e., diameter, viscosity, γ, etc.). $(C_D)_{\text{ave}}$ varies

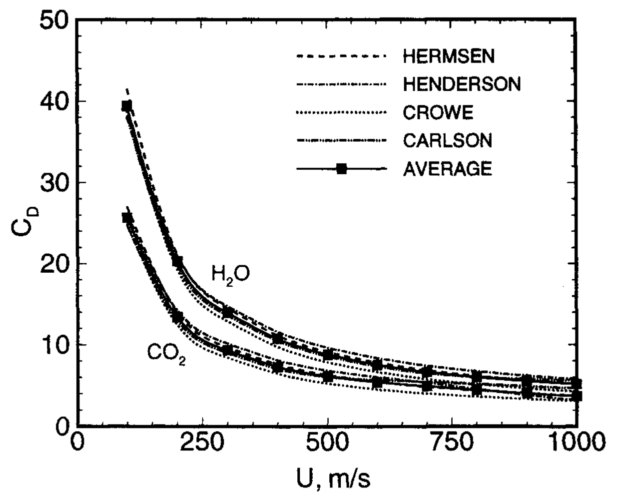

Fig. 1 Comparison of individual drag correlations and $(C_D)_{\text{ave}}$ vs U for CO_2 and H_2O at $Kn = 0.5$ and $T_p = T_g = 2000$ K.

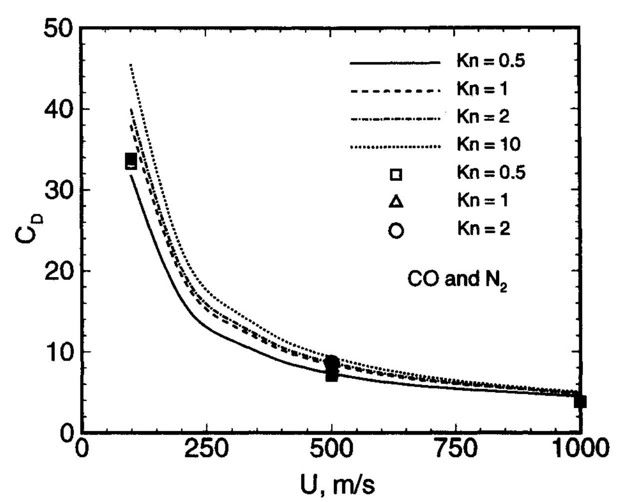

Fig. 2 $(C_D)_{\text{ave}}$ and $(C_D)_{\text{DSMC}}$ vs U for several values of Kn. The symbols are DSMC results: solid = N_2, open = CO.

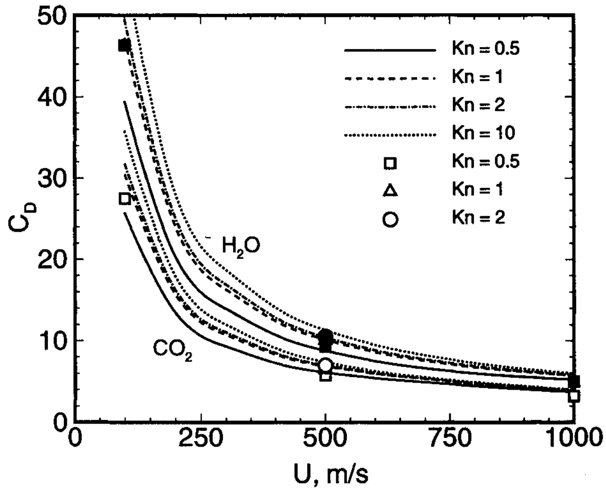

Fig. 3 **$(C_D)_{ave}$ and $(C_D)_{DSMC}$ vs U for several values of Kn. The symbols are DSMC results: solid = H_2O, open = CO_2.**

as the inverse of molecular weight, at a given U; hence, CO_2 has the lowest C_D, and H_2O the highest.

Figures 2 and 3 show that the agreement between $(C_D)_{DSMC}$ and $(C_D)_{ave}$ is quite good at velocities of 500 and 1000 m/s; however, at 100 m/s the agreement degrades slightly. $(C_D)_{ave}$ underpredicts $(C_D)_{DSMC}$ by up to 15% at U = 100 m/s, Kn = 0.5.

Drag in Multicomponent Gas Mixtures

The drag on a particle in a mixture of gases can be written as

$$D = \sum_i \frac{1}{2} \rho_i U^2 A C_{D_i} \tag{8}$$

where i represents the gas. The drag can be written in terms of mass fraction as

$$D = \frac{1}{2} \rho U^2 A \sum_i X_i C_{D_i} \tag{9}$$

to give the total drag on the particle in a mixture of gases. Thus, since the drag correlations represent the C_D of the individual plume gases, they can also be used in situations involving gas mixtures.

Conclusions

$(C_D)_{ave}$ gives good overall agreement with the $(C_D)_{DSMC}$ for CO, N_2, CO_2, and H_2O gases at representative solid rocket plume temperatures. Hence, $(C_D)_{ave}$ can be used in engineering applications to plumes. In applications in which it might not be feasible to use $(C_D)_{ave}$, the Hermsen drag correlation can be used since it is the most accurate of the four empirical correlations.

Acknowledgments

This research was performed under subcontracts A93S-9 and A94S-24 with Sverdrup Technology, Inc., Arnold Air Force Base, Tennessee, and monitored by Robert A. Reed and Robert S. Hiers, III. The authors wish to acknowledge the many helpful discussions about DSMC with Richard G. Wilmoth of NASA Langley Research Center, Hampton, Virginia.

References

[1]Vogenitz, F. W., Bird, G. A., Broadwell, J. E., and Rungaldier, H., "Theoretical and Experimental Study of Rarefied Supersonic Flows About Several Simple Shapes," *AIAA Journal*, Vol. 6, No. 12, 1968, pp. 2388–2394.

[2]Bird, G. A., "Monte-Carlo Simulation in an Engineering Context," *Rarefied Gas Dynamics*, edited by S. S. Fisher, Vol. 74, Pt. I, Progress in Astronautics and Aeronautics, AIAA, New York, 1981, pp. 239–255.

[3]Bird, G. A., *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, Oxford Sciences Publications, Clarendon, Oxford, England, UK, 1994.

[4]Bird, G. A., "The G2/A3 Program System Users Manual," G. A. B. Consulting Pty. Ltd., Version 1.8, Killara, New South Wales, Australia, March 1992.

[5]Hermsen, R. W., "Review of Particle Drag Models," JANNAF Performance Standardizations Subcommittee 12th Meeting Minutes, Chemical Propulsion Information Agency, Jan. 1979; also Nickerson, G. R., Coats, D. E., Hermsen, R. W., and Lamberty, J. T., Jr., "A Computer Program for the Prediction of Solid Propellant Rocket Motor Performance (SPP)," Air Force Rocket Propulsion Lab., AFRPL TR-83-036, Edwards AFB, CA, Sept. 1984, Section 6.3.

[6]Henderson, C. B., "Drag Coefficients of Spheres in Continuum and Rarefied Flows," *AIAA Journal*, Vol. 14, No. 6, 1976, pp. 707, 708.

[7]Crowe, C. T., "Drag Coefficient of Particles in a Rocket Nozzle," *AIAA Journal*, Vol. 5, No. 5, 1967, pp. 1021, 1022.

[8]Carlson, D. J., and Hoglund, R. F., "Particle Drag and Heat Transfer in Rocket Nozzles," *AIAA Journal*, Vol. 2, No. 11, 1964, pp. 1980–1984.

[9]Fields, J. C., "DSMC Analysis of Heat Transfer and Drag on Particles in Solid Rocket Plumes," M.S. Thesis, Dept. of Mechanical and Aerospace Engineering, Univ. of Missouri–Rolla, Rolla, MO, Dec. 1994.

Two-Parameter Wideband Spectral Model for the Absorption Coefficients of Molecular Gases

K. Kamiuto*
Oita University, Oita 870-11, Japan

Introduction

LOW-RESOLUTION spectral models for the absorption coefficients of IR gases are needed to perform efficient spectral computations of radiative transfer in real gases[1] or in gas-particle composite media.[2]

The Edwards model[1] and modified Edwards model[3] derived from the Elsasser regular band model with the exponential band-envelope have been utilized for this purpose. These models can be generally represented in the following form:

$$\kappa_\nu = (\alpha/K_1\omega)\exp[-\Delta\nu/K_1\omega]\tanh K_2\eta \tag{1}$$

Here, α is the integrated band intensity (cm^{-1}/gm^{-2}), ω is the bandwidth parameter (cm^{-1}), η is the line-overlap parameter, and K_1 and K_2 are the parameters. Furthermore, $\Delta\nu$ is defined as follows: $\Delta\nu = \nu_u - \nu$, for an asymmetric band with upper limit ν_u, $\Delta\nu = \nu - \nu_l$, for an asymmetric band with lower limit ν_l, and $\Delta\nu = 2|\nu - \nu_c|$, for a symmetric band with center ν_c. In the expression for $\Delta\nu$, ν is the wave number of radiation (cm^{-1}).

Received Aug. 15, 1994; revision received Nov. 8, 1994; accepted for publication Dec. 19, 1994.

*Professor, High-Temperature Heat Transfer Laboratory, Department of Production Systems Engineering. Member AIAA.